Why study gases?

Continuous collisions of gas particles at the walls of the container generate a resultant force on the walls.

Force per unit area is the pressure exerted by the gas!

\[
p = \frac{F}{A}
\]
Ideal gas law: Beginnings

**Assumptions:**

1. Gases consist of very large number (N) of very small ($v_i$) particles 
   \[ \Sigma v_i = 0 \] ideal gases \> 0 real gases

2. Particles do not attract or repel each other
   (Inter-particle forces do not exist -- ideal gas)

3. Particles are in constant motion, 
   travel in straight lines between collisions, 
   undergo elastic collisions between themselves and the walls 
   (Follow Newtonian Mechanics)

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![Diagram](image.png)

Gas container (V)

Area (A)

Length (l)

Cross-section along X-axis
Let: 
\[ \begin{align*} 
    m & \quad \text{mass of each gas particle} \\
    v & \quad \text{velocity along X coordinate} \\
    l & \quad \text{length of the container along the X coordinate} \\
    N & \quad \text{the total number of particles in the container} \\
\end{align*} \]

the particles travel parallel to the X-coordinate  
(this condition is not necessary but useful now)

(Classical Mechanics)

**Strategy:**

\[ \text{Pressure} = \text{Force} \div \text{Area} \]

\[ \text{Force} = \text{rate of change of momentum} \]

**Approach:** We will compute

\[ \begin{align*} 
    & \Rightarrow \text{change in momentum per collision with the wall} \\
    & \Rightarrow \text{number of collisions per second (one gas particle)} \\
    & \Rightarrow \text{rate of change of momentum of one gas particle} \\
    & \Rightarrow \text{force exerted by one particle and pressure exerted} \\
    & \Rightarrow \text{force exerted by all the particles} \\
    & \Rightarrow \text{Total pressure of the gas on one wall} \\
    & \Rightarrow \text{Total pressure exerted on any wall or at any point} \\
\end{align*} \]
Derivation of \( pV = \frac{1}{3} N m c^2 \)

1. Momentum of the particle before colliding with the wall \( = mv \)
   Momentum after the collision with the wall \( = m(-v) \)
   Change in momentum due to one collision \( = (-mv) - (mv) = -2mv \)

2. Distance traveled by the particle in one second is \( = v \) meters
   Distance of one round trip \( = 2l \) meters
   Number of round trips made by one particle in one s \( = v / 2l \)
   \( = \) number of collisions made by one particle at the wall in 1 s

3. Rate of change of momentum per second for one particle (X-direction)
   \( = \) Change in momentum per collision \( \times \) number of collisions per second
   \( = -2mv \frac{v}{2l} = -\frac{mv^2}{l} \)
4. Force exerted by one particle:
   - Force acting on the particle during the collision
   - Force exerted on the wall by the particle
   - Force per unit area due to one particle
     
     (\(V = \text{total volume of the gas}\))

\[
\begin{align*}
\text{Force per unit area due to one particle} &= \frac{mv^2}{V} \\
\text{Force exerted on the wall by the particle} &= \frac{mv^2}{l} \\
\text{Force acting on the particle during the collision} &= -\frac{mv^2}{l} \\
\end{align*}
\]

This is the pressure exerted by one particle at the wall placed perpendicular to the X-axis.
Pressure due to all particles is...

5. Pressure due to N particles on this wall

\[ p = m \frac{v_1^2 + v_2^2 + v_3^2 + ... + v_N^2}{V} \]

where \( v_1, v_2, .. v_N \) are the individual particle velocities along the X-axis

However, we do not know the individual velocities of the particles!

But, we can express this sum in terms of the average velocity \( <v^2> \) along X-axis as

\[ <v^2> = \frac{v_1^2 + v_2^2 + ... + v_N^2}{N} \]

Or \[ v_1^2 + v_2^2 + ... + v_N^2 = <v^2> N \]

\[ p = m(N<v^2>/V) \]

or \[ pV = Nm <v^2> \]

The X-component of the velocity ‘v’ is related to the average square speed as:

\[ \langle c^2 \rangle = \langle u^2 \rangle + \langle v^2 \rangle + \langle w^2 \rangle = 3 \langle v^2 \rangle \]

Since, \[ \langle u^2 \rangle = \langle v^2 \rangle = \langle w^2 \rangle \] (gas is not flowing)

where \( u, v \) and \( w \) are the velocity components along the X, Y, and Z directions

\[ pV = \frac{1}{3} Nm \langle c^2 \rangle \]
\[ p \, V = \frac{1}{3} \, N \, m \, \langle c^2 \rangle \]

**Triumph of the Kinetic Theory of Gases!**

Pressure of the gas is related to:

- number of particles in the container, (increases with number of moles)
- mass of the particles, heavier particles may exert higher pressure
- average squared speed of the gas particles can be obtained from pressure

If the particles are not moving parallel to the X-axis, we can write the three perpendicular components of the velocity vectors as \( u, v, \) and \( w, \) and the same expression is obtained. We did use these components!

Shape of the vessel is irrelevant, as the pressure is the same in all directions and pressure (\( p \)) is an isotropic property, independent of the direction.

Thus, the assumptions we made are completely justifiable!
**P, V and T Relation for gases**

**Boyle's Law:** Pressure and volume of the gas are related as $pV = \text{constant}$

$= (1/3) Nm\langle c^2 \rangle$, 
Is a constant at constant temperature and gas mass

Temperature of the gas is related to its velocity as:

Average kinetic energy ($<\text{KE}>$) of the gas $= N\langle \epsilon \rangle = N[(1/2) m\langle c^2 \rangle]$

But, $<\text{KE}> = (3/2) N (kT) = (1/2) N m\langle c^2 \rangle$ where $k$ is the Boltzmann constant

Or $N m\langle c^2 \rangle = (3/2)(2/1) NkT$

$pV = (1/3) Nm\langle c^2 \rangle = (1/3) (3NkT) = nN_A kT = nRT$ 

$n = \text{mole number}$

Therefore, $pV = nRT$

$N_A = \text{Avagadro's number}$

$R = (N_A k) = \text{gas constant}$
Pressure or $V$ and $T$ are related as

$$pV = n \, R \, T$$

$pV = \text{constant}$ \quad \text{Boyle’s law}

$V = \text{constant} \times T$ \quad \text{Charles Law}

Figure out the units of $R$ and its value using STP conditions!
Charles' Law  V and temperature (t °C) of the gas are related as

\[ V = a + b \, t \]  

at constant pressure and mass

\[ V = V_o + \left( \frac{\partial V}{\partial t} \right)_p \, t \]

\[ V_o = \text{volume of the gas at zero °C}. \]

But compressibility factor alpha, \( \alpha_o \), is a constant for all gases.

\[ \alpha_o = \frac{1}{V_o} \left( \frac{\partial V}{\partial t} \right)_p \]

So that,  
\[ V = V_o + V_o \alpha_o \, t = V_o \alpha_o \left( \frac{1}{\alpha_o} + t \right) \]

\( \frac{1}{\alpha_o} = 273.15 \)

OR  
\[ V = V_o \alpha_o \, T \]

Where  
\[ T = (273.15 + t), \]

Temperature (T) of the gas is the absolute scale of temperature  (Kelvin scale)

What happens to \( V \) when \( T = 0? \)